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Quark and Nucleon Self-Energy in Dense Matter

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Abstract

In a recent work we introduced a nonlocal version of the Nambu–Jona-Lasinio(NJL) model that was designed to generate a quark self-energy in Euclidean space that was similar to that obtained in lattice simulations of QCD. In the present work we carry out related calculations in Minkowski space, so that we can study the effects of the significant vector and axial-vector interactions that appear in extended NJL models and which play an important role in the study of the ρ, ω and a_1 mesons. We study the modification of the quark self-energy in the presence of matter and find that our model reproduces the behavior of the quark condensate predicted by the model-independent relation $\langle\bar{q}q\rangle_\rho = \langle\bar{q}q\rangle_0(1-\sigma_N\rho_N/f_\pi^2m_\pi^2+\cdots)$, where σ_N is the pion-nucleon sigma term and ρ_N is the density of nuclear matter. (Since we do not include a model of confinement, our study is restricted to the analysis of quark matter. We provide some discussion of the modification of the above formula for quark matter.) The inclusion of a quark current mass leads to a second-order phase transition for the restoration of chiral symmetry. That restoration is about 80% at twice nuclear matter density for the model considered in this work. We also find that the part of the quark self-energy that is explicitly dependent upon density has a strong negative Lorentz-scalar term and a strong positive Lorentz-vector term, which is analogous to the self-energy found for the nucleon in nuclear matter when one makes use of the Dirac equation for the nucleon. In this work we calculate the nucleon self-energy in nuclear matter using our model of the quark self-energy and obtain satisfactory results in agreement with the values of the scalar and vector nucleon potentials in matter found in either theoretical or phenomenological studies.

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I. INTRODUCTION

In recent years there has been a great deal of interest in understanding the properties of quark matter at relatively low temperature and high density [1-5]. Since it is difficult to study QCD at finite chemical potential using lattice simulations, the model of choice for such studies has been the Nambu–Jona-Lasinio model [6-8]. Of particular interest is the prediction of diquark condensates and color superconductivity at high densities. (Such studies may be relevant to the properties of neutron stars.) Calculations made for dense matter using the NJL model are carried out in Minkowski space. These calculations are limited in that they do not include a model of confinement and are, therefore, unable to provide a comprehensive description of the hadronic phase present at low density and temperature. However, it is generally believed that a good deal of information may be gained by studying quark matter, with a proper study of the hadronic phase deferred until some future time.

As is well known, the NJL model provides a microscopic dynamical description of chiral symmetry breaking with the generation of associated quark vacuum condensates and constituent masses. In the standard versions of the NJL model [6-8], the constituent quark mass that is generated in the model is a constant. However, it is known from lattice simulations of QCD that the constituent mass goes over to the current quark mass when the quark momentum, p^2, is less than about -2 GeV^2 [9]. It is our belief that, if we are to use the NJL model to study dense matter, it is desirable to make the model as realistic as possible. To that end, we have introduced a nonlocal version of the NJL model [10] that is able to reproduce the Euclidean-space behavior of the quark mass seen in lattice simulations of QCD [9]. To carry out that program we have introduced a momentum-dependent $q\bar{q}$ interaction in the calculation of the quark self-energy and have separated the regularization of the model from the specification of that interaction. (This procedure requires the introduction of additional parameters into the model.)

Recently, we have seen an attempt to obtain the quark self-energy in Minkowski space by analytic continuation of a Euclidean-space form based upon gluon exchange enhanced at small momentum transfers to simulate confinement [11]. The resulting Minkowski-space values exhibit resonant-like structures and very large values of the constituent mass. Such results do not appear to have any natural physical interpretation.

We note that the quark self-energy may be written as

$$\Sigma(k^2) = A(k^2) + B(k^2)\not{k} \tag{1.1}$$

in vacuum. In matter there is another four-vector, η^μ, that describes the motion of the matter rest frame. It is useful to put $\eta^2 = 1$ and to note that, if we work in the matter rest frame, we can put $\eta^\mu = [1, 0, 0, 0]$. In matter, we have

$$\Sigma(k^2, \eta \cdot k) = A(k^2, \eta \cdot k) + B(k^2, \eta \cdot k)\overline{\not{k}} + C(k^2, \eta \cdot k)\not{\eta}\,, \tag{1.2}$$

where we have found it useful to introduce the four-vector

$$\overline{k^\mu} = k^\mu - (k \cdot \eta)\eta^\mu\,. \tag{1.3}$$

Note that $\overline{k^0} = 0$ in the matter rest frame. In that frame, we may write

$$\Sigma(k^0, \vec{k}) = A(k^0, \vec{k}) - B(k^0, \vec{k})\vec{\gamma} \cdot \vec{k} + \gamma^0 C(k^0, \vec{k})\,. \tag{1.4}$$

As we will see, A and B satisfy coupled nonlinear equations, while C may be calculated independently. (Note that A and C have the same dimension, while B is dimensionless.)

We first discuss our results for the case in which we neglect the dependence of A, B, and C on k^0. We anticipate that the dependence on k^0 will be relatively weak and justify that assumption later in this work. As a further simplification, we will at first neglect B and study the behavior of $A(\vec{k}, \rho)$, where ρ is the density of quark matter which is taken to contain equal numbers of up and down quarks. In this case, the maximum value of $A(\vec{k}, \rho)$ is found at $\vec{k} = 0$, leading to a simpler presentation of our results. We then go on to the consideration of the coupled equations for $A(\vec{k}, \rho)$ and $B(\vec{k}, \rho)$. As usual, we may introduce a density and momentum-dependent mass defined by

$$M(\vec{k}, \rho) = \frac{A(\vec{k}, \rho)}{1 - B(\vec{k}, \rho)}\,. \tag{1.5}$$

We provide values of $A(\vec{k}, \rho)$, $M(\vec{k}, \rho)$ and $C(\vec{k}, \rho)$ in the following sections.

We note that the inclusion of $C(\vec{k}, \rho)$ precludes the passage to Euclidean space. The analogous problem arises when one introduces a finite chemical potential. As we will see, $C(\vec{k}, \rho)$ is quite large at finite density and can only be neglected at very small densities. In vacuum, Lorentz invariance leads to a relation between $C(k^2)$ and $B(k^2)$. However, our formalism does not maintain Lorentz invariance. (For example, our regulator depends only upon $|\vec{k}|$.) Therefore, in the following we will only calculate the contribution to $C(\vec{k},\rho)$ from the matter, which takes the form of two Fermi seas of up and down positive-energy quarks with Fermi momentum k_F.

The Lagrangian of our model is

$$\begin{aligned}\mathcal{L} = \;& \bar{q}(i\partial\!\!\!/ - m^0)q + \frac{G_S}{2}\sum_{i=0}^{8}[(\bar{q}\lambda^i q)^2 + (\bar{q}i\gamma_5\lambda^i q)^2] \\ & -\frac{G_V}{2}\sum_{i=0}^{8}[(\bar{q}\lambda^i\gamma_\mu q)^2 + (\bar{q}\lambda^i\gamma_5\gamma_\mu q)^2] \\ & +\frac{G_D}{2}\{\det[\bar{q}(1+\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]\} \\ & +\mathcal{L}_{\mathrm{conf}}\,. \end{aligned} \tag{1.6}$$

This Lagrangian was used in Ref. [10] with $G_V = 0$. There, both the condensate values and self-energies were given for the up (down) and strange quarks. For the present work we neglect the 't Hooft interaction ($G_D = 0$) and our model of confinement. We also drop the strange quark from consideration, so that the quark current mass matrix is $m^0 = \mathrm{diag}(m_u^0, m_d^0)$ with $m_u^0 = m_d^0$. Since the 't Hooft interaction contributes to the self-energy [10], its neglect leads us to use a larger value of G_S than that we would use if the 't Hooft interaction were included in our analysis.

The organization of our work is as follows. In Section II we present the coupled nonlinear equations that determine $A(\vec{k},\rho)$ and $B(\vec{k},\rho)$ and also provide an expression for $C(\vec{k},\rho)$, and the density-dependent condensate $\langle\bar{u}u\rangle_\rho = \langle\bar{d}d\rangle_\rho$. We present values of $A(\vec{k},\rho)$ and the condensate, when we neglect $B(\vec{k},\rho)$. In Section III we consider finite values of $B(\vec{k},\rho)$ and provide results for $A(\vec{k},\rho)$, $M(\vec{k},\rho)$, $C(\vec{k},\rho)$ and the condensate. In Section IV, we consider the dependence of A, B and C on k^0 and provide some results for $A(k_0,\vec{k})$ in vacuum. In Section V we use the quark self-energy calculated here to obtain the nucleon self-energy in nuclear matter in a simple model. Finally, Section VI contains some further discussion and conclusions.

II. THE QUARK SELF-ENERGY

In our earlier work we obtained the quark self-energy from the solution of the equation depicted in Fig. 1 [10]. There, the open circle is a momentum-dependent quark interaction,

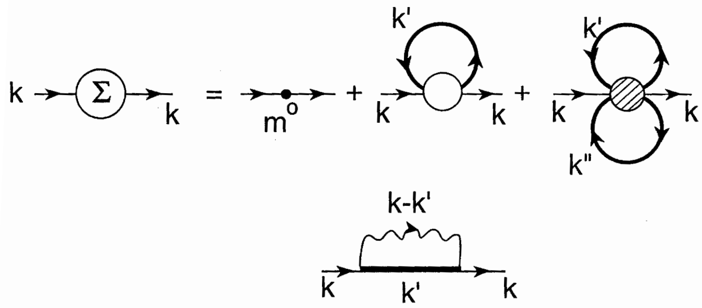

FIG. 1: The equation for the quark self-energy that was solved in Euclidean space in Ref. [10] is shown. Here, m^0 is the current quark mass. The open circle represents the momentum-dependent $q\bar{q}$ interaction of the nonlocal model. The third term on the right-hand side of the figure represents the 't Hooft interaction and the fourth term arises from our model of confinement. The heavy lines are quark propagators which include the self-energy, $\Sigma(k)$, in their definition. [See Eq. (2.1).]

obtained by the replacement

$$G_S \longrightarrow f(k-k')\,G_S f(k-k')\,, \tag{2.1}$$

where k and k' are the quark momenta entering (or leaving) the interaction. We have used

$$f(k-k') = \exp[-(k-k')^{2n}/2\beta] \tag{2.2}$$

with $n=4$ and $\beta = 20$ GeV^8 in Ref. [10]. The corresponding nonlocal Lagrangian is given in Ref. [10]. On the right-hand side of Fig.1, the 't Hooft interaction (third term) and the confinement interaction (fourth term) are neglected for the purposes of this work. In the second term we have contributions from the negative-energy states in vacuum as well as the positive-energy states at finite density, specified by the quark Fermi momentum, k_F, of the up and down quark Fermi seas.

In general, the quark propagator is

$$iS(k, k\cdot\eta) = \frac{i}{\not{k} - \Sigma(k^2, k\cdot\eta) + i\epsilon}, \tag{2.3}$$

which we will write as

$$iS(k^0, \vec{k}) = \frac{i}{(k^0 - C(k^0,\vec{k}))\gamma^0 - (1 - B(k^0,\vec{k}))\vec{\gamma}\cdot\vec{k} - A(k^0,\vec{k}) + i\epsilon}, \tag{2.4}$$

in the matter rest frame. In a first approximation we write

$$iS(k^0, \vec{k}) = \frac{i}{(k^0 - C(\vec{k}))\gamma^0 - (1 - B(\vec{k}))\vec{\gamma}\cdot\vec{k} - A(\vec{k}) + i\epsilon}, \tag{2.5}$$

with $A(\vec{k})$, $B(\vec{k})$ and $C(\vec{k})$ density-dependent, in general. (On occasion we will write $A(\vec{k},\rho)$, etc.) We see that the presence of $C(\vec{k})$ precludes the passage to Euclidean space that was made in Ref.[10]. That is analogous to the problem created by the introduction of a chemical potential in the formalism. Note that

$$\begin{aligned}(k^0 - C(\vec{k}))^2 &- (1 - B(\vec{k}))^2\vec{k}^2 - A^2(\vec{k}) + i\epsilon \\ &= [k^0 - E^+(\vec{k}) + i\epsilon][k^0 - E^-(\vec{k}) - i\epsilon]\end{aligned} \tag{2.6}$$

with

$$E^+(\vec{k}) = C(\vec{k}) + \sqrt{\vec{k}^2(1 - B(\vec{k}))^2 + A^2(\vec{k}^2)} \tag{2.7}$$

and

$$E^-(\vec{k}) = C(\vec{k}) - \sqrt{\vec{k}^2(1 - B(\vec{k}))^2 + A^2(\vec{k}^2)}\,. \tag{2.8}$$

Here $E^+(\vec{k})$ may be interpreted as the (on-mass-shell) energy of the positive-energy states, while $E^-(\vec{k})$ refers to the negative-energy states.

In order to simplify the notation somewhat, we write

$$iS(k^0, \vec{k}) = \frac{i}{\not{\Pi}(k^0,\vec{k}) - A(\vec{k}) + i\epsilon}, \tag{2.9}$$

where we have defined a four-component quantity

$$\Pi^\mu(k^0, \vec{k}) = [k^0 - C(\vec{k}), (1 - B(\vec{k}))\vec{k}]\,. \tag{2.10}$$

We also introduce a scalar quantity

$$\rho_S(\vec{k}) = iN_c(-1)\int\frac{d^4k'}{(2\pi)^4}\,\frac{4A(\vec{k}')f^2(k-k')}{\Pi^2(k'^0,\vec{k}') - A^2(\vec{k}') + i\epsilon}, \tag{2.11}$$

where the minus sign is due to the closed Fermion loop in Fig. 1 and the factor of 4 comes from forming the trace associated with the closed loop. We see that $\rho_S(\vec{k})$ does not depend upon k^0, since we are making an on-shell approximation in the calculation of $f(k-k')$. In vacuum we have

$$-\, iA(\vec{k}) = -im^0 + (2G_S i)\rho_S(\vec{k})\,, \tag{2.12}$$

or

$$A(\vec{k}) = m^0 - 2G_S\rho_S(\vec{k})\,, \tag{2.13}$$

where $\rho_S(\vec{k})$ is real and negative. We remark when $f(k-k')=1$, $A(\vec{k})\to m$ and $\rho_S(\vec{k})\to \langle\bar{u}u\rangle$, so that we regain the usual result [6-8]

$$m_u = m_u^0 - 2G_S\langle\bar{u}u\rangle\,, \tag{2.14}$$

$$m_d = m_d^0 - 2G_S\langle\bar{d}d\rangle\,. \tag{2.15}$$

(Note that our G_S is one-half of the G_S defined in Ref. [7].)

The evaluation of $\rho_S(\vec{k})$ proceeds by closing to contour in the complex k^0 plane. In the vacuum we find

$$\rho_S(\vec{k}) = -2N_c\int\frac{d^3k'}{(2\pi)^3}\,\frac{f^2(k-k')A(\vec{k}')}{\sqrt{\vec{k}'^2(1-B(\vec{k}'))^2+A^2(\vec{k}')}}\,. \tag{2.16}$$

We define

$$E(\vec{k}) = \sqrt{\vec{k}^2(1-B(\vec{k}))^2+A^2(\vec{k})}\,, \tag{2.17}$$

and

$$(k-k')^2 = (E(\vec{k})-E(\vec{k}'))^2 - (\vec{k}-\vec{k}')^2\,, \tag{2.18}$$

so that $f(k-k')$ depends only upon $|\vec{k}|$, $|\vec{k}'|$ and the angle between $\vec{k}$ and $\vec{k}'$.

Integrals such as that in Eq. (2.16) require regularization. For our calculations we insert a factor $\exp[-\vec{k}'^2/\alpha^2]$ with $\alpha = 0.60$ GeV. (We have used the same Gaussian regulator in our calculations of meson spectra, where we have used $\alpha = 0.605$ GeV [12-14]. However, those calculations included a model of confinement, so that we can not directly take over the parameters G_S, G_V and G_D used in those works.) We remark that an expression for the density-dependent condensate $\langle \bar{u}u \rangle_\rho$ may be obtained by using Eq. (2.16) with $f(k-k')=1$.

In the presence of matter it is useful to separate the propagator into two parts, one of which will give rise to the explicitly density-dependent terms. In this regard, it is useful to generalize Eq. (5.8) of Ref. [6]. We define

$$\Lambda^{(+)}(\vec{k}) = \frac{E(\vec{k})\gamma^0 - \vec{\gamma}\cdot\vec{k}(1-B(\vec{k})) + A(\vec{k})}{2A(\vec{k})}\,, \tag{2.19}$$

$$\Lambda^{(-)}(-\vec{k}) = \frac{-E(\vec{k})\gamma^0 - \vec{\gamma}\cdot\vec{k}(1-B(\vec{k})) + A(\vec{k})}{2A(\vec{k})}\,, \tag{2.20}$$

where $E(\vec{k}) = [\vec{k}^2(1-B(\vec{k}))^2 + A^2(\vec{k})]^{1/2}$. Then

$$\begin{aligned} S(k) = \quad & \frac{A(\vec{k})}{E(\vec{k})}\left[\frac{\Lambda^{(+)}(\vec{k})}{k^0 - E^+(\vec{k})} - \frac{\Lambda^{(-)}(-\vec{k})}{k^0 - E^-(\vec{k})}\right] \\ & + \; 2\pi i \frac{A(\vec{k})}{E(\vec{k})}\Lambda^{(+)}(\vec{k})\,\theta(k_F - |\vec{k}|)\,\delta(k^0 - E^+(\vec{k})) \end{aligned} \tag{2.21}$$

In the limit that $A(\vec{k}) \to m^*$ and $B(\vec{k}) = C(\vec{k}) = 0$, we have, with

$$E^*(\vec{k}) = \sqrt{\vec{k}^2 + m^{*2}}\,, \tag{2.22}$$

$$\begin{aligned} S(k) \;=\; & \frac{m^*}{E^*(\vec{k})}\left[\frac{\Lambda^{(+)}(\vec{k})}{k^0 - E^*(\vec{k})} - \frac{\Lambda^{(-)}(-\vec{k})}{k^0 + E^*(\vec{k})}\right] \qquad (2.23) \\ & + \; 2\pi i \frac{m^*}{E^*(\vec{k})}\Lambda^{(+)}(\vec{k})\,\theta(k_F - |\vec{k}|)\,\delta(k^0 - E^*(\vec{k})) \\ =\; & \frac{\not{k} + m^*}{k^2 - m^{*2}} + \frac{i\pi}{E^*(\vec{k})}(\not{k} + m^*)\,\theta(k_F - |\vec{k}|)\,\delta(k^0 - E^*(\vec{k}))\,, \qquad (2.24) \end{aligned}$$

which agrees with Eq. (5.8) of Ref. [6].

In the presence of matter, we have

$$A(\vec{k},\rho) = m^0 - 2G_S[\rho_S^{\mathrm{vac}}(\vec{k}) - \rho_S^{\mathrm{mat}}(\vec{k})]\,, \tag{2.25}$$

where $\rho_S^{\mathrm{mat}}(\vec{k})$ is calculated in the same manner as $\rho_S^{\mathrm{vac}}(\vec{k})$, except that the upper limit of the integral over $|\vec{k}'|$ is k_F. Equation (2.21) is a generalization of a corresponding equation that may be found in Klevansky's review. (See Eqs. (5.18) of Ref. [6].)

If we neglect $B(\vec{k})$, Eqs. (2.11) and (2.16) provide a nonlinear equation for $A(\vec{k})$ which may be solved by iteration. The results of such a calculation are reported in Table I where, for nuclear matter, we put $k_F = 0.268$ GeV. In Figs. 2 and 3 we show values of the condensate $\langle \bar{u}u \rangle$ and $A(0,\rho)$ as a function of the density. These results may be usefully discussed in terms of the relation [15]

$$\langle \bar{u}u \rangle_\rho = \langle \bar{u}u \rangle_0 \left(1 - \frac{\sigma_N \rho_N}{f_\pi^2 m_\pi^2} + \cdots \right) . \tag{2.26}$$

Here σ_N is the pion-nucleon sigma term and ρ_N is the density of nucleons. If we put $\sigma_N = 0.050$ GeV, $\rho_N = (0.109\,\mathrm{GeV})^3$, $f_\pi = 0.0942$ GeV and $m_\pi = 0.138$ GeV, we find a 38% reduction of the condensate at nuclear matter density, which agrees with our results given in Table I and Fig. 2. It is of interest to note that the linear dependence on the density implied by Eq. (2.26) appears to be valid up to about twice nuclear matter density. However, one may be concerned that, since we study quark matter rather than nuclear matter, Eq. (2.26) may not be appropriate. Consider, however, the relation

$$\langle \bar{u}u \rangle_\rho = \langle \bar{u}u \rangle_0 \left(1 - \frac{\sigma_q \rho_q}{f_\pi^2 m_\pi^2} + \cdots \right) , \tag{2.27}$$

where σ_q is the quark sigma term and ρ_q is the number density of the quarks, which we may put equal to $3\rho_N$. Thus, if $3\sigma_q = \sigma_N$, we may use Eq. (2.26). The fact that $\sigma_q \simeq 15$ MeV has been discussed by Vogl and Weise [7]. We have also discussed this matter in great detail in Ref. [16], where we calculated similar values of σ_q using the standard version of the NJL model. We conclude that the use of Eq. (2.26), or Eq. (2.27), with an appropriate value of σ_q, is satisfactory. For example, if $\sigma_q = \sigma_N/3$, as suggested in Ref. [7], the two relations imply the same density dependence of the condensate.

In Fig. 3 we show $A(0,\rho)$ which is the density-dependent mass parameter of the theory when $B(\vec{k},\rho) = 0$. We see that $A(0,\rho)$ follows the trend seen in Fig.2 for the density dependence of the condensate.

III. LORENTZ-VECTOR TERMS OF THE QUARK SELF-ENERGY

We now write $\Sigma(\vec{k}) = \Sigma_S(\vec{k}) + \Sigma_V(\vec{k})$ where

$$\Sigma_V(\vec{k}) = -\vec{\gamma}\cdot\vec{k}B(\vec{k}) + \gamma^0 C(\vec{k}) . \tag{3.1}$$

For the calculation of $C(\vec{k})$ we obtain the contribution from the last term in Eq. (2.21), with the result that

$$C(\vec{k}) = 2G_V \rho_2^V(\vec{k}) \tag{3.2}$$

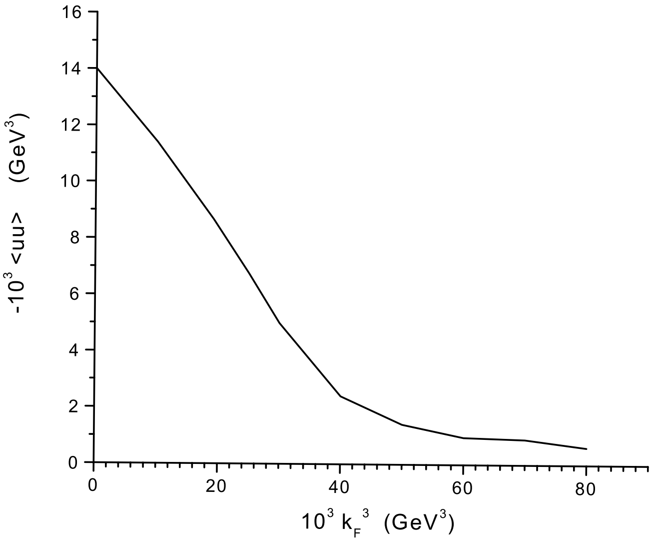

FIG. 2: Values of the condensate $\langle \bar{u}u \rangle$ are given as a function of $10^3 k_F^3$. For nuclear matter $10^3 k_F^3 = 19.2$ GeV^3. [See Table I.] Here $G_S = 13.0$ GeV^{-2} and $B(\vec{k}, \rho)$ is put equal to zero.

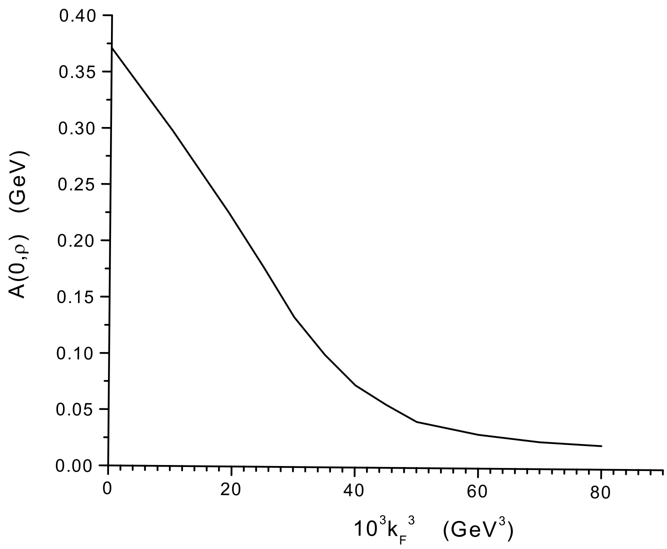

FIG. 3: Values of $A(0, \rho)$ are given as a function of $10^3 k_F^3$. [See Table I and the caption of Fig. 2.]

with

$$\rho_2^V(\vec{k}) = 2N_c \int^{k_F} \frac{d^3k'}{(2\pi)^3} f^2(k - k'). \tag{3.3}$$

$10^3 k_F^3$ (GeV^3)	$-\langle \bar{u}u\rangle^{1/3}$ (GeV)	$-10^3\langle \bar{u}u\rangle$ (GeV^3)	$A(0,\rho)$ (GeV)
0	0.241	14.0	0.371
10	0.225	11.4	0.298
19.2(nm)	0.205	8.67	0.226
25	0.189	6.75	0.177
30	0.171	5.00	0.133
40	0.134	2.46	0.077
50	0.113	1.44	0.041
60	0.0990	0.967	0.030
70	0.0915	0.898	0.024
80	0.0853	0.620	0.021
90	0.0805	0.522	0.018

TABLE I: Values of the condensate and $A(0,\rho)$ are given for $G_S = G_V = 13.0\ \mathrm{GeV}^{-2}$ and $m_u^0 = m_d^0 = 0.005$ GeV. Here $k_F = 0.268$ GeV for nuclear matter. We note the reduction of the condensate of 38% and a 40% reduction of $A(0,\rho)$ at nuclear matter density $[10^3 k_F^3 = 19.2\ \mathrm{GeV}^3]$. Here $\alpha = 0.60$ GeV is used in the Gaussian regulator $\exp[-\vec{k}^2/\alpha^2]$.

An expression for $B(\vec{k})$ may be found from the relation

$$-i[-\vec{\gamma}\cdot\vec{k}B(\vec{k})] = (-2G_V i)(-1)i\int\frac{d^4k'}{(2\pi)^4}\,S(k')f^2(k-k') \tag{3.4}$$

if we only keep the term proportional to $\vec{\gamma}\cdot\vec{k}'$ in the expression for the quark propagator. In vacuum we may compare corresponding terms in Eq. (3.4)

$$-\vec{\gamma}\cdot\vec{k}B(\vec{k}) = -2G_V\int\frac{d^4k'}{(2\pi)^4}\,\frac{-\vec{\gamma}\cdot\vec{k}'[1-B(\vec{k}')]f^2(k-k')}{[k_0'-E^+(\vec{k}')+i\epsilon][k_0'-E^-(\vec{k}')-i\epsilon]}\,. \tag{3.5}$$

Thus,

$$B^{\mathrm{vac}}(\vec{k}) = 2G_V\rho_1^{\mathrm{vac}}(\vec{k})\,, \tag{3.6}$$

with

$$|\vec{k}|\rho_1^{\text{vac}}(\vec{k}) = 2N_c\int\frac{d^3k'}{(2\pi)^3}\,\frac{|\vec{k}'|(\hat{k}\cdot\hat{k}')f^2(k-k')[1-B(\vec{k}')]}{\sqrt{\vec{k}'^2[1-B(\vec{k}')]^2+A^2(\vec{k}')}}\,. \tag{3.7}$$

Here, $\hat{k}$ and $\hat{k}'$ are unit vectors. As in the calculation of $A(\vec{k})$, Eq. (3.6) is generalized to read

$$B(\vec{k}) = 2G_V[\rho_1^{\text{vac}}(\vec{k}) - \rho_1^{\text{mat}}(\vec{k})]\,, \tag{3.8}$$

where $\rho_1^{\text{mat}}(\vec{k})$ is calculated using Eq. (3.7) with an upper limit on $|\vec{k}'|$ of k_F.

In Table II we present results of our calculation of the condensate, $A(0,\rho)$, $A(0,\rho)/[1-B(0,\rho)]$, $C(0,\rho)$, $B(0,\rho)$ and $A(0,\rho)-A(0,0)$. We also define

$$U_S(\vec{k},\rho) = A(\vec{k},\rho) - A(\vec{k},0)\,, \tag{3.9}$$

where $U_S(\vec{k},\rho)$ is the density-dependent modification of $A(\vec{k},0)$ in matter.

It may be seen from the values given in Table II that there is a thirty percent reduction of the condensate at the density of nuclear matter, while the value of $A(0)$ is reduced by thirty-nine percent. We would obtain a thirty percent reduction of the condensate if $\sigma_N = 39$ MeV.

In Figs. 4 and 5 we exhibit values of $A(\vec{k},\rho)$ and $A(\vec{k},\rho)/[1-B(\vec{k},\rho)]$ for various densities and in Fig. 6 we present values of $C(\vec{k},\rho)$. Figure 7 shows the values of $U_S(\vec{k},\rho_{NM})$ and $C(\vec{k},\rho_{NM})$.

IV. OFF-MASS-SHELL EFFECTS

In our calculations we have neglected the k_0 dependence of A, B and C. In this Section we provide some justification of that approximation. We can combine Eqs. (2.9) and (2.11) to yield a nonlinear equation for $A(k^0,\vec{k})$ in vacuum

$$A(k^0,\vec{k}) = -(2G_Si)N_c(-1)\int\frac{d^4k'}{(2\pi)^4}\,\frac{4A(k'^0,\vec{k}')f^2(k-k')}{\not{k}'-A(k'^0,\vec{k}')+i\epsilon}\,. \tag{4.1}$$

Here we have neglected m^0 and $B(k^0,\vec{k})$. We see that k^0 dependence arises from $f^2(k-k')$ which is given by

$$f^2(k-k') = \exp[-(k-k')^{2n}/\beta]\,, \tag{4.2}$$

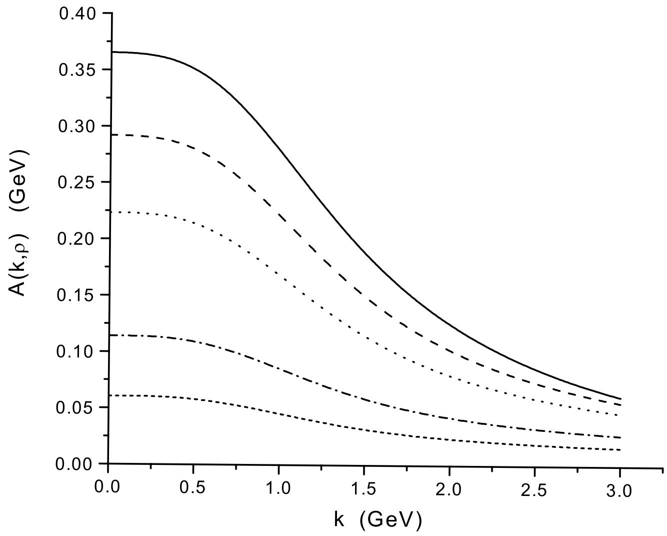

FIG. 4: Values of $A(\vec{k},\rho)$ are given as a function of $|\vec{k}|$ for various densities: a) $10^3 k_F^3 = 0$ [solid line]; b) $10^3 k_F^3 = 10.0$ GeV^3 [dashed line]; c) $10^3 k_F^3 = 19.2$ GeV^3 [dotted line]; d) $10^3 k_F^3 = 30.0$ GeV^3 [dot-dash line] and e) $10^3 k_F^3 = 40.0$ GeV^3 [short dash]. Here $G_S = 13.5$ GeV^{-2}, $G_V = 10.0$ GeV^{-2}, $m^0 = 0.005$ GeV and $\alpha = 0.60$ GeV. [See Table II.]

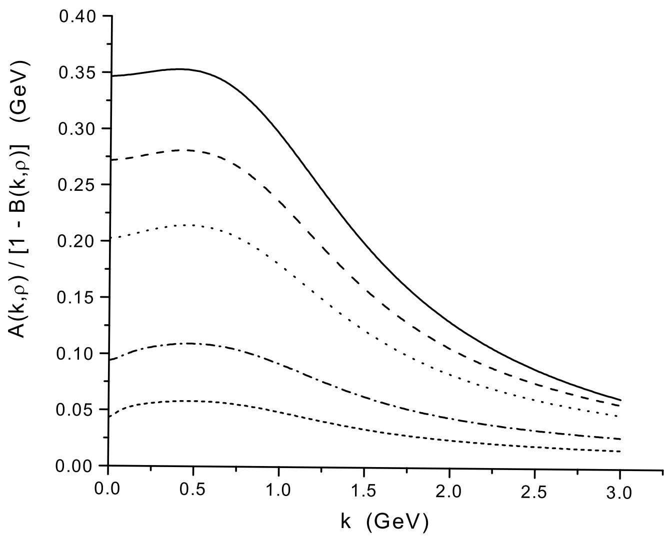

FIG. 5: The quantity $A(\vec{k},\rho)/[1 - B(\vec{k},\rho)]$, which plays the role of a momentum- and density-dependent mass parameter, is shown. [See Table II and the caption of Fig. 4.]

$10^3 k_F^3$ (GeV^3)	$-\langle\bar{u}u\rangle^{1/3}$ (GeV)	$-10^3\langle\bar{u}u\rangle$ (GeV^3)	$A(0,\rho)$ (GeV)	$\frac{A(0,\rho)}{1-B(0,\rho)}$ (GeV)	$U_S(0,\rho)$ (GeV)	$C(0,\rho)$ (GeV)	$B(0,\rho)$
0	0.2401	13.85	0.365	0.347	0	0	-0.0544
10	0.2279	11.84	0.292	0.272	-0.073	0.0794	-0.0736
19.2	0.2128	9.64	0.223	0.203	-0.142	0.0988	-0.1019
30	0.1755	5.36	0.114	0.0942	-0.252	0.115	-0.210
40	0.1441	2.99	0.0606	0.0435	-0.304	0.126	-0.394
50	0.1291	2.15	0.0432	0.0279	-0.322	0.136	-0.545

TABLE II: Various values are given for the case $G_S = 13.5$ GeV^{-2}, $G_V = 10.0$ GeV^{-2}, $m_u^0 = m_d^0 = 0.005$ GeV and $\alpha = 0.60$ GeV. Note a reduction of 30% for the condensate and 39% for $A(0,\rho)$ at nuclear matter density, where $k_F = 0.268$ GeV and $10^3 k_F^3 = 19.2$ GeV^3.

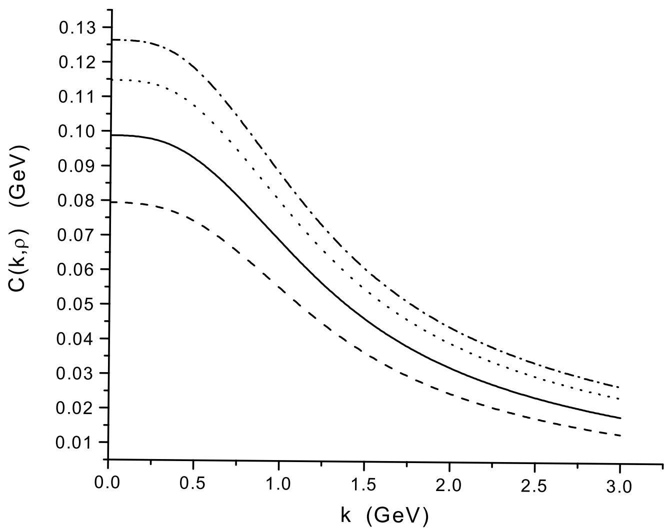

FIG. 6: Values of $C(\vec{k},\rho)$ are shown for various densities: a) $10^3 k_F^3 = 10.0$ GeV^3 [dashed line]; b) $10^3 k_F^3 = 19.2$ GeV^3 [solid line]; c) $10^3 k_F^3 = 30.0$ GeV^3 [dotted line] and d) $10^3 k_F^3 = 40.0$ GeV^3 [dash-dot line]. Here $G_S = 13.5$ GeV^{-2}, $G_V = 10.0$ GeV^{-2}, $m^0 = 0.005$ GeV and $\alpha = 0.60$ GeV.

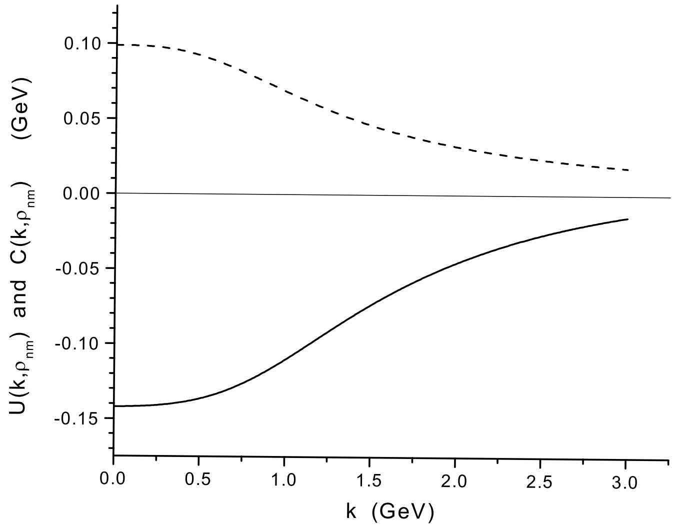

FIG. 7: The values of $U(\vec{k},\rho_{NM}) = A(\vec{k},\rho_{NM}) - A(\vec{k},0)$ [solid line] and $C(\vec{k},\rho_{NM})$ [dashed line] are shown. $U(\vec{k},\rho_{NM})$ represents the density-dependent correction to the vacuum value of the scalar term of the quark self-energy.

with

$$(k-k')^2 = (k^0 - k'^0)^2 - (\vec{k} - \vec{k}')^2 \,, \tag{4.3}$$

which differs from the on-mass-shell version given in Eq. (2.18).

The solution of Eq. (4.1) for $A(k^0,\vec{k})$ is shown in Fig. 8 for various values of k^0. It is seen that the dependence on k^0 is weak as was assumed in this work.

V. THE NUCLEON SELF-ENERGY IN MATTER

If one uses the Dirac equation to describe the interaction of a nucleon with a nucleus, or with nuclear matter, it is found that a strong scalar attraction is needed as well as a strong vector repulsion [17, 18]. The scalar field is of the order of -400 MeV and the vector field is about 300 MeV. It is of interest to see if the nucleon self-energy, $\Sigma_N = V_S + \gamma^0 V_V$, can be calculated in terms of the quark self-energy obtained in this work. To carry out this program we use a simple model of the nucleon in which a quark is coupled to a scalar

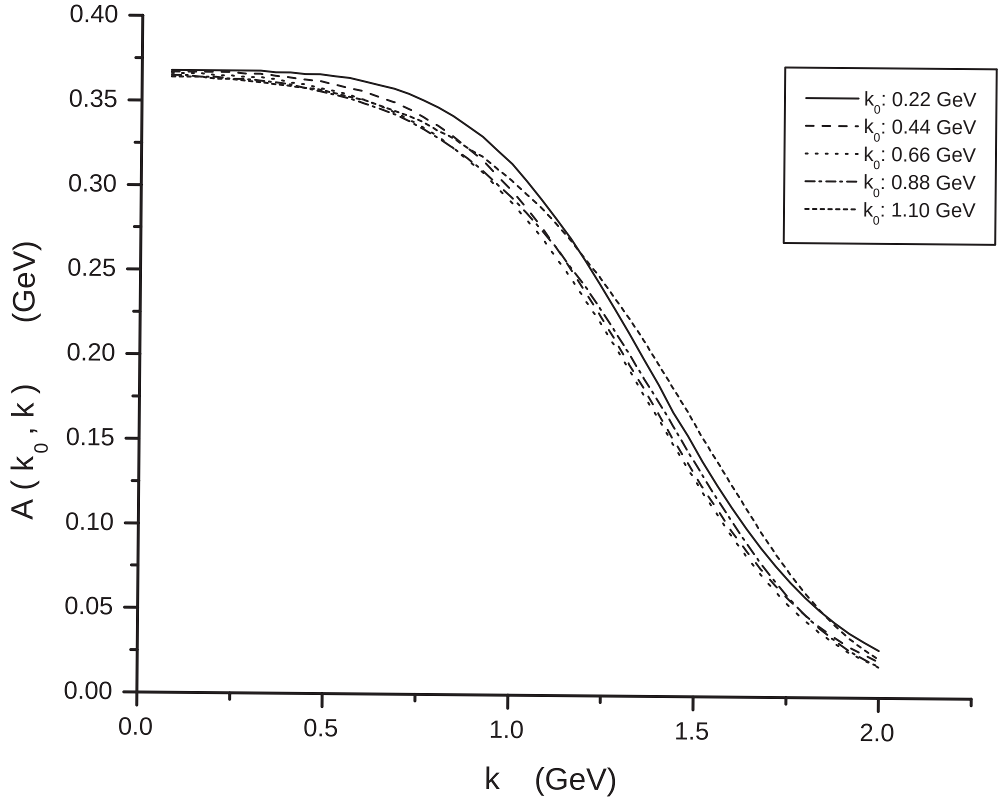

FIG. 8: Values of $A(k^0, \vec{k})$ are shown as a function of $|\vec{k}|$ for various values of k^0.

diquark. The full complexity of the wave function, including vector diquarks and various relativistic effects, is discussed in Ref. [19].

We can calculate the nucleon self-energy in nuclear matter using a triangle diagram in which one of the lower two vertices of the triangle represents a vertex function for a zero-momentum nucleon to emit a quark of momentum $\vec{k}$ leaving a spectator (on-mass-shell) diquark of momentum $-\vec{k}$. The other lower vertex represents the inverse process. At the upper vertex we insert the quark self-energy, $\Sigma(\vec{k}) = U_S(\vec{k}) + \gamma^0 C(\vec{k})$ calculated in this work and integrate over $\vec{k}$. We make use of Fig. 4 of Ref. [19] and parametrize the product of the vertex function and the quark Greens function by the quark-diquark wave function

$$\psi(\vec{k}) = \frac{1}{\sqrt{N}}\, e^{-\vec{k}^2/\lambda^2} u(\vec{k}, s)\,, \tag{5.1}$$

where $u(\vec{k}, s)$ is a spinor for a quark of momentum $\vec{k}$ and spin projection s [19].

The normalization for a single quark is obtained from the relation

$$\frac{1}{N}\int \frac{d^3k}{(2\pi)^3}\, e^{-2\vec{k}^2/\lambda^2} \left(1 + \frac{\vec{k}^2}{[E_q(\vec{k}) + m_q]^2}\right) = 1\,, \tag{5.2}$$

where we put $\lambda = 0.18$ GeV to correspond to the results of Ref. [19]. We may then relate the nucleon self-energy to the quark self-energy. For a nucleon of momentum $\vec{P} = 0$, we have, with $E_q(\vec{k}) = [\vec{k}^2 + m_q^2]^{1/2}$, and $m_q = 0.364$ GeV,

$$V_V = \frac{3}{N}\int \frac{d^3k}{(2\pi)^3} C(\vec{k})\, e^{-2\vec{k}^2/\lambda^2} \left(1 + \frac{\vec{k}^2}{[E_q(\vec{k}) + m_q]^2}\right) , \tag{5.3}$$

and

$$V_S = \frac{3}{N}\int \frac{d^3k}{(2\pi)^3} U_S(\vec{k})\, e^{-2\vec{k}^2/\lambda^2} \left(1 - \frac{\vec{k}^2}{[E_q(\vec{k}) + m_q]^2}\right) . \tag{5.4}$$

The difference of sign in the brackets appearing in Eqs. (5.3) and (5.4) is due to the different behavior of the Dirac matrices, $\mathbf{1}$ and γ^0, at the upper vertex of the triangle.

Since the momentum content of the quark-diquark wave function is small [19], we expect that $V_S \simeq 3U_S(0)$ and $V_V \simeq 3C(0)$, so that $V_V \simeq 296$ MeV and $V_S \simeq -426$ MeV. A more careful evaluation of the integrals in Eqs. (5.3) and (5.4) yields $V_V = 295$ MeV and $V_S = -392$ MeV, which is in general accord with the values given in Refs. [17] and [18].

VI. DISCUSSION

The behavior of matter at high density and low temperature has received a good deal of attention in the last few years [1-5]. A large part of the work in this area has made use of the NJL model. In our work we have attempted to modify the NJL model so that its predictions are in greater accord with QCD. As a first step, in Ref. [10] we introduced a momentum-dependent $q\bar{q}$ interaction which allowed us to reproduce the Euclidean-space behavior for the constituent quark mass obtained in lattice simulation of QCD [9]. In the present study we have considered the important vector interactions of an extended NJL model. Of particular interest is the behavior of the quark condensate at finite density. We find that the linear behavior in the density exhibited in Eq. (2.26) holds in our model up to about twice nuclear matter density. Also, we note that the use of a nonzero current quark mass is important for that result, since in the absence of an explicit chiral symmetry breaking term, the model exhibits a first-order phase transition at about 1.25 times nuclear matter density.

Another point of interest are the results shown in Fig. 7. The quark self-energy is similar to that found in relativistic nuclear physics, with strong scalar attraction and strong vector repulsion. The simple calculation reported in Section V suggests that the quark self-energy, when multiplied by 3, provides a satisfactory estimate of the nucleon self-energy which is in accord with results of relativistic nuclear physics [17, 18].

There is a body of work based upon the solutions of the Schwinger-Dyson and Bethe-Salpeter equations with a phenomenological form for the gluon propagator [20-23]. This body of work is reviewed in Ref. [24]. In contrast to the results of our work, it is found that the quark condensate *increases* with increasing chemical potential. The authors of Ref. [20] argue that the baryon density is zero up to the critical chemical potential, μ_c, for deconfinement. They state that "This result is an expected consequence of confinement which entails that each additional quark must be locally paired with an antiquark thereby increasing the density of condensate pairs as μ is increased. For this reason, as long as $\mu < \mu_c$, there is no excess of particles over antiparticles in the vacuum and hence the baryon number density remains zero." We note, however, that the baryon density usually considered in such calculations is due to the presence of *nucleons*, which have a finite baryon density. As noted earlier, in our work the baryon density is due to the presence of Fermi seas of up and down quarks, which serve to provide a model of the baryon density that would arise due to the presence of nucleons. We suggest that the conclusions presented in Ref. [20] do not refer to the situation of physical interest in which one studies the value of the quark condensate in nuclei or in nuclear matter.

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